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Abstract

An optimal tension regulator system is designed for a steel strip winding process. A simplified dynamic model of the winding process contains a parameteric nonlinearity due to variations of the coil radius. A parameter imbedding technique is applied to derive the optimum feedback control for a range of parameter values. Simulation results are given for an implementation of the regulator system designed using the numerical values from a real plant.

I. INTRODUCTION

In this paper an attempt is made to apply optimal control theory to the design of a feedback control system for a strip winding process. Strip winding processes appear in a wide variety of industrial plants, such as paper mills, aluminum and steel mills, plastic webb-production, high speed handling of magnetic tapes, etc. The common part of different strip winding processes is given in Figure 1. (See Appendix 1) The strip is coming from part A of the plant with velocity v_s. The winding reel winds it with angular velocity $\Omega = n\omega$ where n is the gear ratio and ω is the angular speed of the shaft of the DC drive motor.

In many applications the main objective is to maintain the tension, T, at a desired constant value. Large variations of the strip velocity v_s and the coil radius r make the tension regulation a difficult task for a single-input single-output regulator system. For this reason the regulator system has a more complicated structure. The motor armature voltage e_a and field voltage e_f are both used as control inputs and several process variables are used as outputs. The angular speed Ω is measured by a tachogenerator and the tension T is measured by a tensiometer, while additional information is obtained by measuring some other variables, such as the coil radius r, the armature current i_a etc. [1-4].

It is convenient to separately consider problems of two different stages of the winding process: <u>constant speed winding</u>, and <u>variable speed winding</u>.

In the constant speed winding the problem is to maintain the tension T and the coil periferral speed v_c constant for the entire range of values of the slowly varying radius r. If the friction losses and the dynamics due to the slow variation of r are neglected, the equations of the constant speed winding are $v_c = r\Omega$ and $M = nrT$, where M is the motor torque. A fairly common constant speed control law which can keep T and v_c constant in spite of the changes of r, is to maintain i_a = const and $\frac{\phi}{r}$ = const where ϕ is the magnetic flux of the motor excitation field [1-3]. This is usually done by using two feedback loops. In the first loop current feedback is used to control e_a. In the second loop the error signal $M - kr = e_m$ is used to control ϕ, that is e_f. Another approach to the constant speed winding is to control the tension by varying the position of a specially built pair of rolls ("briddles"),[5].

The second and more difficult problem is to maintain constant tension during the variable speed winding and in particular during the acceleration at the start, and the deceleration at regular or emergency stops. During these periods tension T may oscillate and even break the material [4]. This deteriorates the quality of the strip and causes serious losses in material and production time, which is especially important in expensive cold steel strip mills.

In this paper a model of the strip winding process is derived which makes it possible to use the theory of the optimum linear systems with quadratic performance indices [6]. Since the coil radius is considered as a parameter rather than as a state variable, the nonlinearity caused by its variations is reduced to a parametric one. Then an imbedding procedure is used to obtain the parameter dependent optimum feedback control [7].

The method is applied to a steel strip winding process. To make analysis and design results more realistic, all the numerical values are taken from a temper mill installation of the Armco Steel Corporation.

2. MODEL OF THE PROCESS

In this section a model of the winding process shown in Figure 1 is derived under the assumption that the dynamics due to slow variations of r and some small time constants can be neglected [2-8]. All the quantities used in the derivation are defined in Appendix 2. The torque equation at the motor shaft is

$$J\dot{\omega} = M - B\omega - nrT \tag{1}$$

where $J = J_m + n^2 J_L$, $B = B + n^2 B_L$, $M = K_1 \phi i_a$, and a dot denotes differentiation with respect to time. Neglecting the armature inductance the equation of the armature circuit is $e_a = R_a i_a + K_2 \phi \omega$. The force balance equation for the elastic strip (Hook's law) is

$$T = C\int_o^t (v_c - v_s)dt + T_o \tag{2}$$

where v_s is a function of the tension and the rolling mill velocity. This function depends on the specific mill stand and it is usually determined empirically [9,10]. Let v denote v_s when there is no tension, T = 0. When the tension is greater than zero, it pulls the strip out and hence v_s is greater than v. It is assumed here that v_s increases linearly with T,

$$v_s = v(1 + sT) \tag{3}$$

where s is an empirical "slipping" coefficient [2,3].

To formulate our control problem we first rewrite the model (1), (2), and (3) in standard state variable form. Since it is convenient to have all the state variables directly measurable, it seems natural to select Ω and T as the state variables. Using (1) and the expressions for M, Ω and i_a,

$$\dot{\Omega} = -\frac{1}{J}\left(B + \frac{K_1 K_2 \phi^2}{R_a}\right)\Omega - \frac{r}{J} T + \frac{nK_1\phi}{JR_a} e_a. \tag{4}$$

Differentiating (2) and using (3),

$$\dot{T} = Cr\Omega - CsvT - Cv. \tag{5}$$

Although the mathematical model (4) and (5) seems to be a simplified description of the process in Figure 1, it can illustrate basic steps of the design procedure presented in this paper. Except for an increase of computational difficulties, the procedure would remain the same for a more realistic fourth or fifth order model which would include armature circuit and tensiometer dynamics.

The control problem for the system (4) and (5) can be stated as follows. The armature voltage e_a and the field voltage e_f are selected as control variables. The velocity v is an external disturbance and the radius r is a slowly varying parameter. A regulator is to be designed which will keep the tension T close to a desired value for all the values of r from a given range and for several typical disturbances v(t). Both r and v can be measured during the process and this information may be used in the regulator design.

3. THE REGULATOR PROBLEM

Since the variations of r are slow we assume that the equality $\frac{\phi_o}{r_o} r - \phi = 0$ is maintained by acting on the control variable e_f as in existing installations [1-3]. Thus we consider that the only control variable is e_a and we use it for high speed regulation of the tension T. Since $K_1\phi = \frac{K_1\phi_o}{r_o} r$ and $K_2\phi = \frac{K_2\phi_o}{r_o} r$, we let $K_1\phi_o = K_i$ and $K_2\phi_o = K_b$. Then (4) and (5) can be rewritten as follows

$$\dot{\Omega} = -\frac{1}{J}\left(B + \frac{K_i K_b}{R_a r_o^2} r^2\right)\Omega - n^2 \frac{r}{J} T + \frac{nK_i r}{Jr_o R_a} e_a \tag{6}$$

$$\dot{T} = Cr\Omega - Csv(t)T - Cv(t). \tag{7}$$

For computational convenience introduce the normalized variables

$$x_1 = \frac{\Omega - \Omega_i}{\Omega_d}, \quad x_2 = \frac{T - T_n}{T_n} \text{ and } u = \frac{e_a - e_a^o}{e_a^o} \tag{8}$$

where T_n is the desired value of T and $\Omega_i = \frac{v(t)}{r} \cdot (1 + sT_n)$, and when v = const, then $\Omega_i = \Omega_d$. In terms of the normalized variables the state equation is

$$\dot{x} = A(r,t)x + B(r)u + D(r,t) \tag{9a}$$

or

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11}(r) & a_{12}(r) \\ a_{21} & a_{22}(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1(r) \\ 0 \end{bmatrix} u + \begin{bmatrix} d_1(r,t) \\ 0 \end{bmatrix} . \qquad (9b)$$

The expressions for elements of A, B, and D matrices are given in Appendix 3. It is pointed out that the coefficients a_{11}, a_{12}, and b_1 are functions of the slowly varying parameter r and that a_{22}, which depends on v(t), becomes a function of time in the variable speed winding. The disturbance d_1 is a function of both r and t, but in the constant speed winding it is the function of the radius r only.

The state equation (9b) is now rewritten assuming a realistic value for slipping coefficient. Using $sT_n = 0.1$ and taking the other numerical values from Table 1 we obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3.43\zeta & -5.26\zeta \\ 261 & -.39v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 7.0\zeta \\ 0 \end{bmatrix} u + \begin{bmatrix} -1.70\delta\zeta \\ 0 \end{bmatrix} \qquad (10)$$

where

$$\zeta = \zeta(r) = r^2/(12.745 + 6.4\, r^4) \qquad (11)$$

$$\delta = \delta(t) = -1.03 + .0296\, v(t). \qquad (12)$$

The range of variations of r is 1-3.0 feet. The function v = v(t) assumed in the design is given in Figure 2. Thus δ(t) is known and an open loop compensation of this disturbance is possible.

Table 1

Armco Steel Corporation No. 6 Temper Winding Mill Specifications

I. Mill Data	
Maximum Winding Reel Diameter, $2r$	72.0 in
Minimum Winding Reel Diameter, $2r_o$	24.0 in
Maximum Steel Thickness, h	0.1196 in
Minimum Steel Thickness	0.0149 in
Maximum Steel Width, d	72.0 in
Minimum Steel Width	24.0 in
Distance--Mill-to-Winding Reel, L	30.0 ft
Work Roll Diameter	20.0 in
Back-up Roll Diameter	30.0 in
Strip Steel Velocity	200-600 ft/min
Inertia of the Winding Reel, J_L	22,000 lb ft^2
II. Winding Reel Motor Data	
Horsepower, hp (3 motors)	1500 hp
Speed, ω	300-1200 rpm
Inertia of the Motor, J_m	12,745 lb ft^2
Voltage (back emf) at max. Speed, e_b	500 volts
Rated Armature Current, i_a (per motor)	840 amps
Shunt Field Resistance, R_f	2.76 ohm
Shunt Field Inductance, L_f	9.5 henries
Armature Resistance, R_a	0.0182 ohms
Armature Inductance, L_a	0.000814 henry
Motor Gear Ratio, $n:1$	1:1.85
Motor Torque constant at $r = r_o$, k_i	6.78 lb/ft/amp

We let the control u be $u = u_f + u_\delta$ and obtain u_δ from $7.0\ \zeta u_\delta - 1.7\ \delta\zeta = 0$. Hence $u_\delta = .242\ \delta$. Then the disturbance term is eliminated from the state equation,

$$\begin{bmatrix} \dot{x}_1 \\ \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3.43\zeta & -5.26\zeta \\ \\ 261 & -.39V \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \\ x_2 \end{bmatrix} + \begin{bmatrix} 7.0\zeta \\ \\ 0 \end{bmatrix} u_f. \quad (13)$$

To determine u_f we introduce a quadratic performance index which penalizes the error in tension and excessive use of control

$$J = \tfrac{1}{2} \int_o^{tp} (x'Qx + u_f' R u_f)\,dt \quad (14)$$

where t_p is the duration of the process, $R = 1$ and $Q = \begin{bmatrix} 0 & 0 \\ 0 & q_2 \end{bmatrix}$ and prime denotes a transpose. The parameter q_2 is a weighting factor determining the relative importance of the tension error and energy expenditure.

The design problem can now be stated as follows: design a regulator which measures the states x_1 and x_2 (and, if necessary the parameter r and the disturbance v), and generates a control u_f such that J is minimum for every pair of initial conditions $x_1(0)$ and $x_2(0)$, for every value of the radius r, $1 \leq r \leq 3$, and for v(t) defined in Figure 2.

4. REGULATOR DESIGN

We consider first the constant speed winding and let in (14) $t_p = \infty$. It is well known [6] that the control u_f which minimizes (14) for a given q_2 and all r is

$$u_f = -R^{-1}B'(r)\, K(r)\, x(r,t). \tag{15}$$

K(r) is the steady state value of the solution of matrix Riccati equation

$$\dot{K}(r) = -A'(r)K(r) - K(r)\, A(r) + K(r)\, S(r)\, K(r) - Q \tag{16}$$

and $S(r) = B(r)\, R^{-1}B'(r)$ and $K(r)\,\big|_{t=t_p} = 0$. It appears that the function K(r) has to be obtained by a sequence of solutions of (16) for different values of r, $1 \leq r \leq 3$. If in addition to this we take into account that several trials may be necessary for a proper choice of the weighting factor q_2, it becomes clear that even in this second order problem such an approach requires excessive amount of computation. To avoid this difficulty we apply an imbedding technique described in [7]. In this technique (16) is solved only for $r = r_o$. Then the value $K(r_o)$ is used as an initial condition to solve the imbedding equation

$$\frac{dK}{dr}(A-SK) + (A-SK)'\frac{dK}{dr} = -K\alpha - \alpha'K + K\beta K \tag{17}$$

whose solution is K(r) with $\alpha = \frac{dA}{dr}$ and $\beta = \frac{dS}{dr}$. In our problem (17) has the following scalar form:

$$\begin{bmatrix} \frac{dK_{11}}{dr} \\ \frac{dK_{12}}{dr} \\ \frac{dK_{22}}{dr} \end{bmatrix} = - \begin{bmatrix} (3.re+49\xi K_{11})\xi & -261 & 0 \\ (5.26+49\xi K_{12})\xi & 3.43\xi+49\xi^2+.39V & -261 \\ 0 & 5.26+49\xi K_{12} & .39V \end{bmatrix}^{-1} \begin{bmatrix} 49\eta\xi K_{11}^2+3.43\eta K_{11} \\ 98\xi\eta K_{11}K_{.2}+ \\ 3.43\eta K_{12}+5.26\eta K_{11} \\ 49\xi\eta K_{12}^2+5.26\eta K_{12} \end{bmatrix} \tag{18}$$

where

$$\xi = \xi(r) = r^2/(12.745 + 6.4r^4)$$

$$\eta = \eta(r) = \frac{d\xi(r)}{dr} = (25.49r^{-3} - 12.8r) \, .\xi^2(r).$$

For several values of q_2, this equation is used to obtain the feedback gains $f_1(r)$ and $f_2(r)$ defined by (15) and (19),

$$u_f = -f_1x_1 - f_2x_2. \tag{19}$$

The above technique can alternatively be used to improve the choice of q_2 for a given r in which case the imbedding equation has the following form,

$$\frac{dK}{dq_2} (A-SK) + (A-SK)' \frac{dK}{dq_2} = -\sigma \tag{20}$$

where $\sigma = \frac{dQ}{dq_2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Using (20) the feedback gains f_1 and f_2 are also obtained as functions of q_2 for several values of r. The result is given in Fig. 3. The optimum system responses for different values of q_2 and r are given in Fig. 4. Finally, the feedback gains f_1 and f_2 corresponding to $q_2 = 10^6$ are selected for the constant speed winding.

Next we consider the variable speed winding which can occur at any r, $1 \leq r \leq 3$. At this stage of the process the disturbance function v(t) appears as a command and is assumed as in Fig. 2. Since the control u_δ provides an open-loop compensation of this disturbance, the regulator problem for the variable speed winding will differ from the constant speed problem only due to the presence of time varying a_{22} element of system matrix A in (9) and the finite integration limit t_p in (14). Therefore one can start with the value of q_2 selected in the constant speed case. The Riccati equation (16) with the time varying coefficients must be used. Solving (16) with $q_2 = 10^6$ and for r = 1,2, and 3 for both acceleration and deceleration it has been found out that the resulting time varying feedback gains can be approximated by the functions f_1 and f_2 obtained for constant speed problem in Figure 3. Typical responses obtained are given in Figure 5. For the sake of comparison the responses for $q_2 = 10^3$ are also shown.

An implementation of the control system obtained by the above design is shown in Figure 6. The open loop control u_δ is obtained by direct measurement of the mill-speed v(t) by the tachogenerator No. 2.

In the feedback part of the regulator the radius-depending gain $f_1(r)$ is approximated by a second order curve $f_1(r) = a_o + a_1 r + a_2 r^2$ which is implemented using a radius transducer and multiplier π_2. The relation (20) is then implemented by the tachogenerator No. 1, tensiometer and multiplier π_1, (note that f_2 is a constant).

Several simulation tests are made with the model of the whole system. Typical acceleration-constant widing-deceleration stages of the process is shown in Figure 7, where it is assumed that there were two impulse disturbances present between the three stages.

5. CONCLUSIONS

The results obtained in this paper for a simplified model of the strip winding process indicate that the linear-quadratic state regulator theory and the imbedding procedure may be used to design the control system for a more realistic higher order model of the same process. In a higher order model the dynamics due to the variations of the coil radius and the coil moment of inertia, as well as the time constants of the tensiometer should be taken into account. The singular perturbation method [11,12] could be applied in which case this the solution obtained here can be used as the low-order nominal solution.

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Appendix 1: Figures

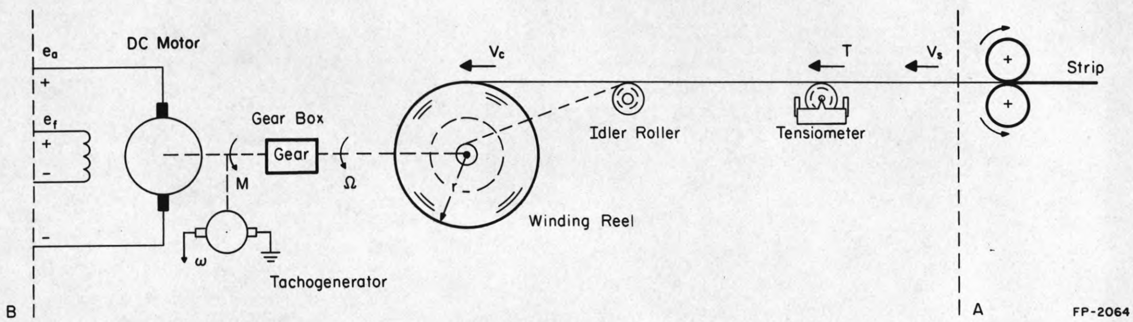

Figure 1. A strip winding process.

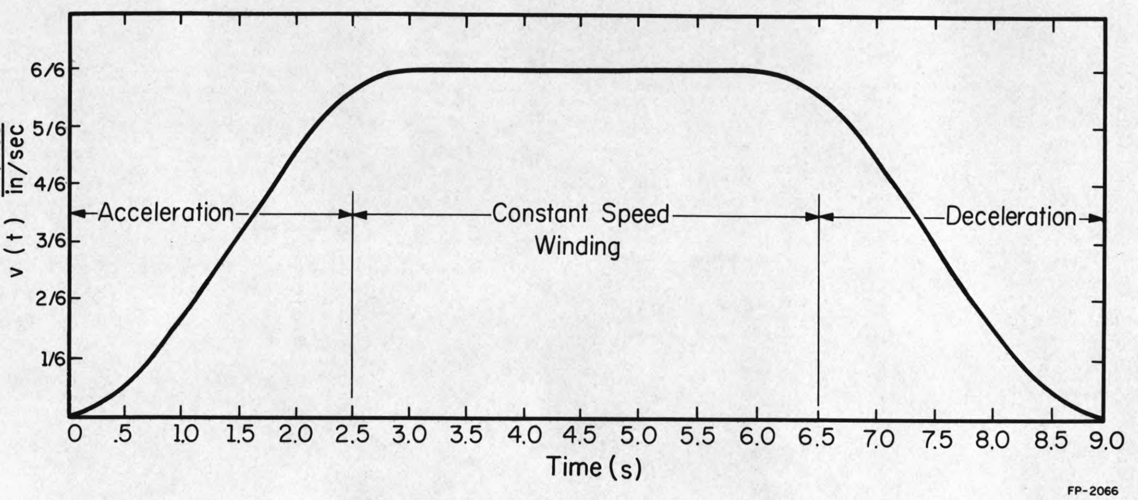

Figure 2. Strip disturbance velocity,
v(t) versus time.

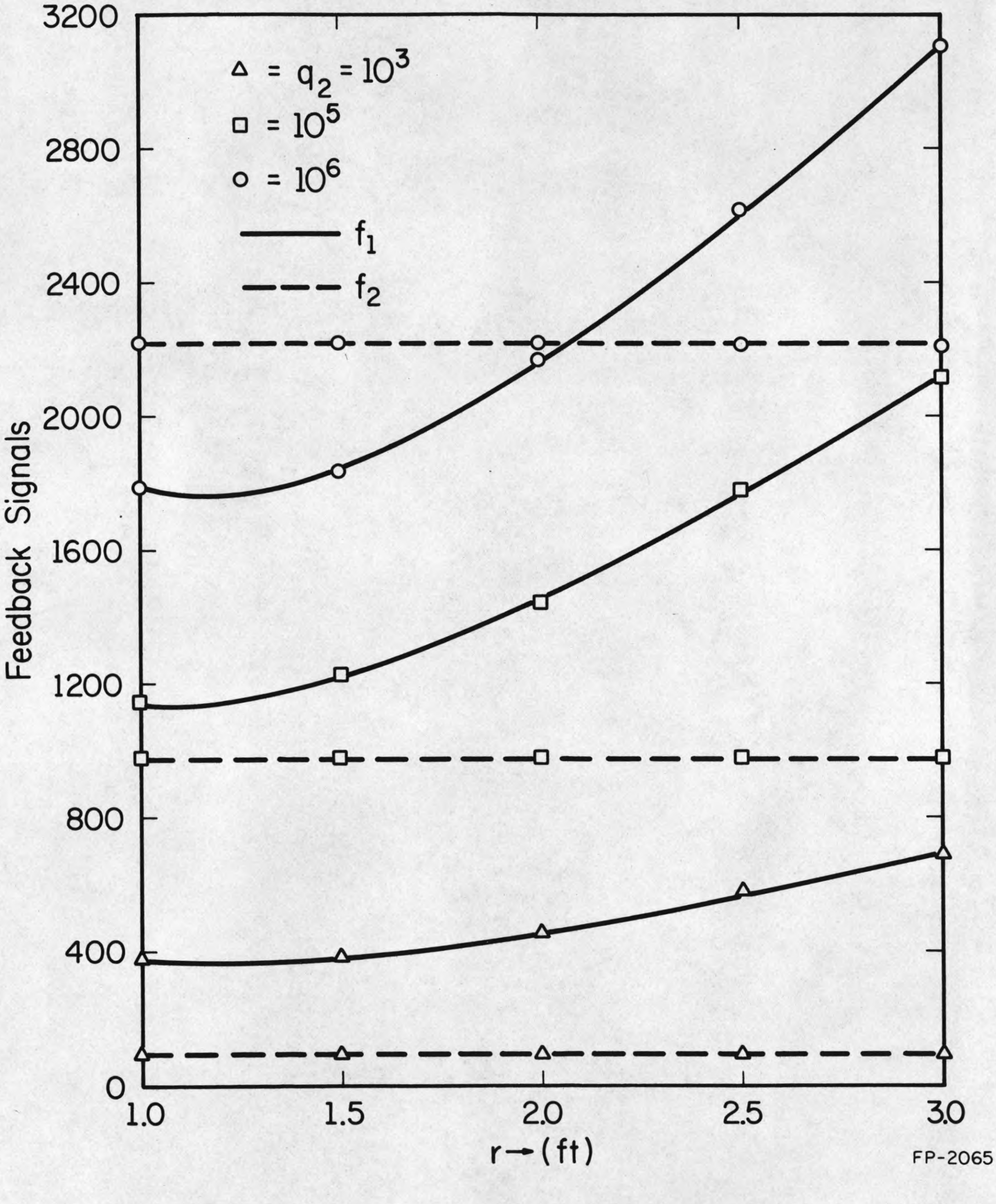

Figure 3. Feedback signals, $f_1(r)$ and $f_2(r)$ vs. radius for three values of q_2.

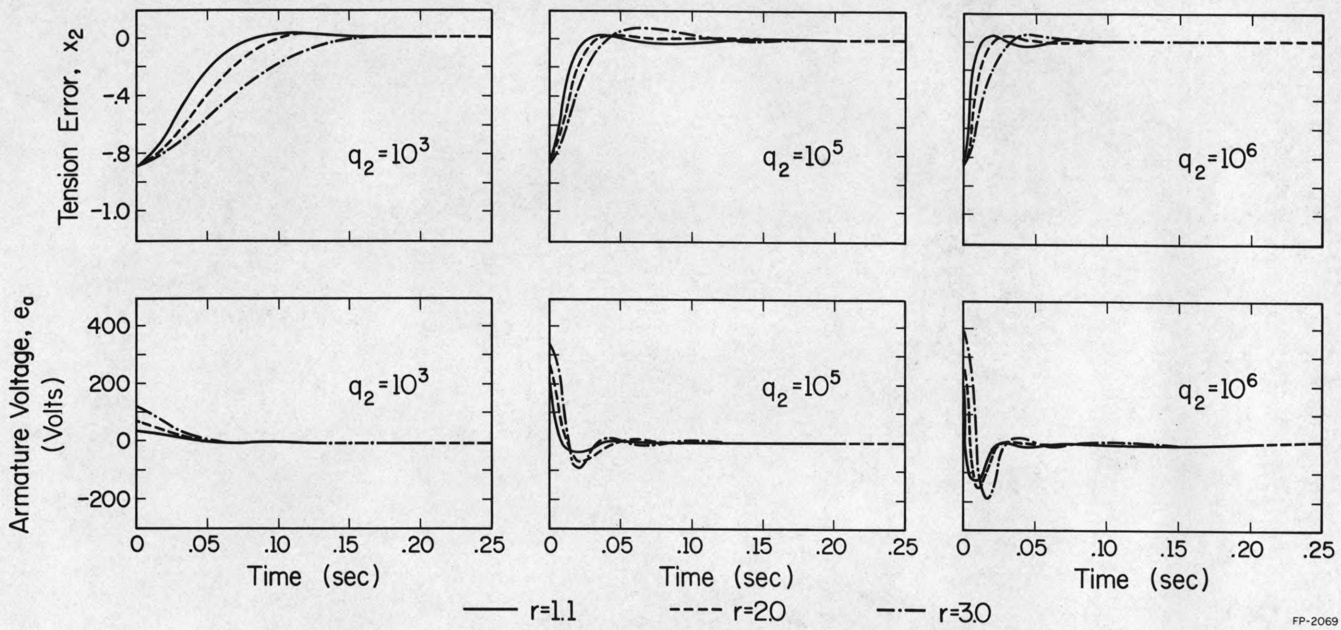

Figure 4. Optimum system responses for constant winding process.

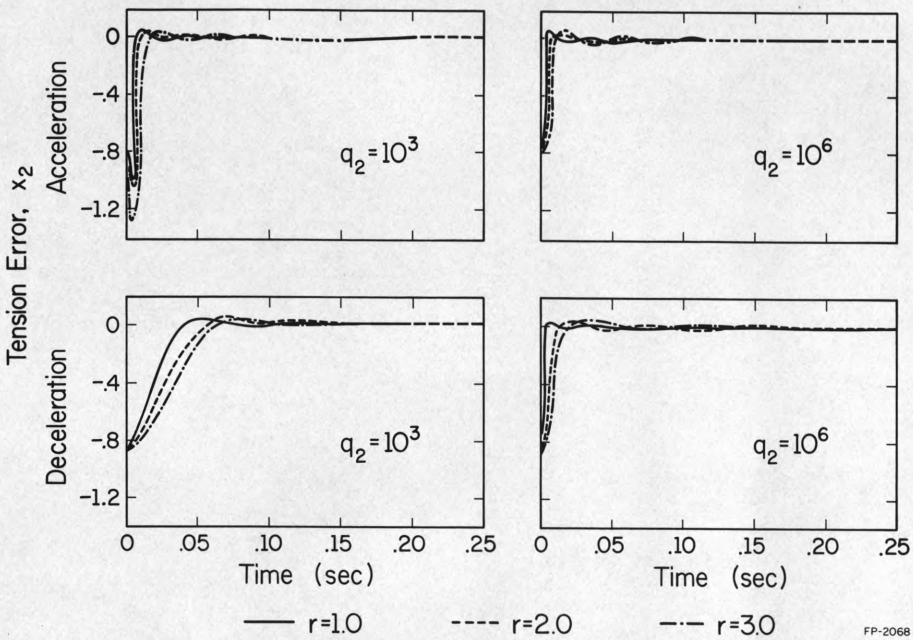

Figure 5. Optimum system responses for acceleration and deceleration processes.

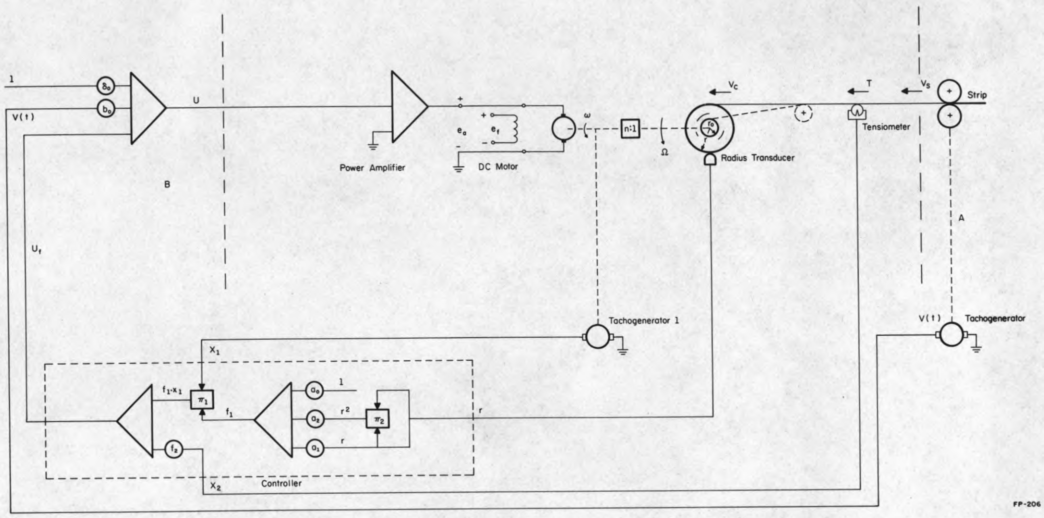

Figure 6. An implementation of the controller.

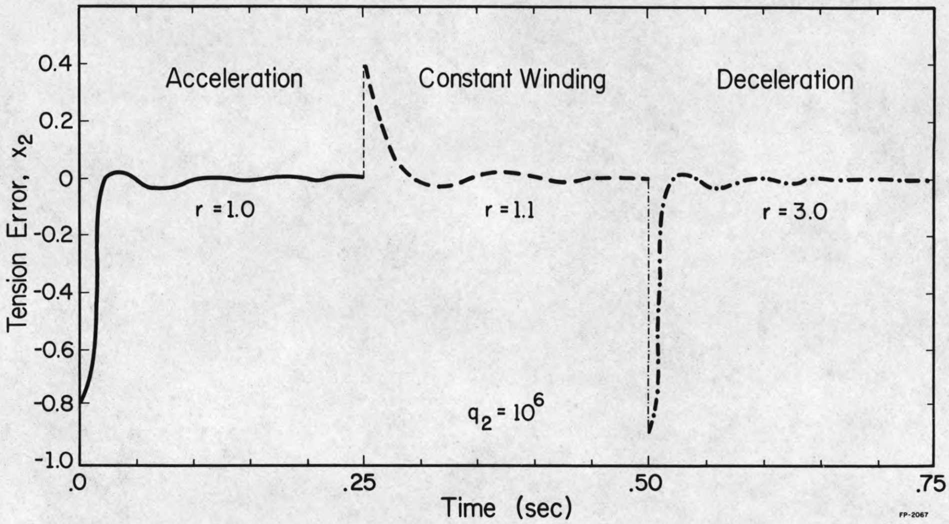

Figure 7. Responses for a typical winding process.

Appendix 2: Nomencluture

v_s = strip velocity at the output of stand

Ω = tension reel angular velocity

n = gear box turns ratio

ω = motor shaft angular velocity

T = tension in the strip

M = motor torque

e_a = motor armature voltage

e_f = motor field voltage

r = tension reel radius (slow variable parameter)

i_a = motor armature current

v_c = strip velocity at the tension reel (coil periferral speed)

ϕ = motor magnetic flux webers

J = equivalent moment of inertia, motor-winding reel $(J_m + n^2 J_L)$

B = equivalent friction loss motor-winding reel $(B_m + n^2 B_L)$

K_1 = a known constant

R_a = armature resistance

K_2 = a known constant

C = strip coefficient of elasticity

T_o = initial value of tension in strip

s = slipping coefficient

v = strip velocity at zero tension (mill speed)

r_o = initial value of winding reel radius

ϕ_o = motor flux at $r = r_o$

K_i = motor torque constant

K_b = motor back emf constant speed

Ω_i = ideal angular velocity of winding reel (variable speed)

Ω_d = desired angular velocity of winding reel (constant speed)

T_n = nominal tension

e_a^o = desired initial value of armature voltage

u_f = closed loop control

u_δ = open loop control

K = Riccati matrix, $K = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix}$

x = state variables vector

u = control signal

d = maximum steel width (length of winding reel)

ρ = density of coiler material

Note that the term "winding reel" and "coiler" are interchanged in parts of this paper.

Appendix 3

The elements of Matrices A, B, and D can be obtained directly from equations (6, 7, and 8),

$$a_{11}(r) = -\frac{K_i K_B r^2}{R_a\left(J_m + \frac{n^2}{2}\,\pi d\rho \quad r^4\right)} = -\frac{K_i K_b r^2}{R_a\left(J_m + G_L r^4\right)}$$

G_L is a constant, d and ρ are length and density of the winding reel, respectively.

$$a_{12}(r) = -\frac{n^2 T_n r}{\Omega_d\left(J_m + G_L \; r^4\right)}$$

$$a_{21} = \frac{Cv_n}{T_n}$$

$$a_{22}(t) = - C\,\varsigma\, v(t)$$

$$b_1(r) = \frac{nK_i e_a^o \;.\; r^2}{R_a \Omega_d\left(J_m + G_L \; r^4\right)}$$

$$d_1(r,t) = \frac{nrK_i e_a^o}{R_a J \,\Omega_d} - \frac{n^2 T_n}{J\,\Omega_d} - \frac{K_i K_b}{R_a J}\,\frac{\Omega_i}{\Omega_d}\, r^2$$

$$= \frac{r^2}{J\,(r)}\;\frac{nK_i e_a^o}{R_a v_n} - \frac{n^2 T_n}{v_n} - \frac{K_i K_b\,(1+ST_n)}{R_a v_n}\, v(t)$$

$$= \delta(t)\;\zeta(r)$$

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An optimal tension regulator system is designed for a steel strip winding process. A simplified dynamic model of the winding process contains a parameteric nonlinearity due to variations of the coil radius. A parameter imbedding technique is applied to derive the optimum feedback control for a range of parameter values. simulation results are given for an implementation of the regulator system designed using the numerical values from a real plant.

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